9/13/17 "Some people ask "Why?", I ask "Why not?"." -Anonymous
HW: "Multiplying Probability" homework section

AIM: How do we multiply probabilities?

Warm Up:

The results of a survey of the student body at Central High School about television viewing preferences are shown below.

	Comedy Series	Drama Series	Reality Series	Total
Males	95	65	70	230
Females	80	70	110	260
Total	175	135	180	490

Are the events "student is a male" and "student prefers reality series" independent of each other? Justify your answer.

$$P(\text{Male}) \cdot P(\text{reality}) = P(\text{BOTH})$$

$$\frac{230}{490} \cdot \frac{180}{490} = \frac{70}{490}$$

$$\frac{41400}{240100} = \frac{70}{490}$$

$$.17 \neq .14$$

Dependent

HW: #1-5 (HW see)
Independence Pkt

1. In each of the following, a scenario is given with two events. Explain whether these events are independent or dependent.

(a) A coin is flipped and lands on a head. The coin is flipped a second time and lands on its head again. Is the probability of it landing on heads the second time dependent on it landing on head the first time? Explain.

No, these two are independent events. The probability of the coin landing on heads the second time doesn't in any way depend on how it landed the first time.

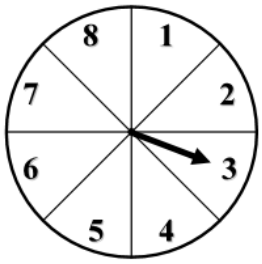
(b) An elementary class consists of 8 boys and 10 girls. A child is chosen at random and it is a girl. A second child is randomly chosen again from the remaining children and it is a boy. Was the probability of choosing the boy dependent on choosing a girl first? Explain.

Yes, it certainly was. Since the first child was not replaced, the gender of the child chosen first alters the probability of the probability of a boy being picked second. So, these are not independent.

2. A newspaper did a survey of adults and found that 54% of the population as a whole favored stricter gun control laws. They broke down the results along gender lines and found that 65% of women favored stricter laws while only 44% of men favored them. If a person was selected at random, are the events of being a woman and being in favor of stricter gun control laws dependent or independent? Explain.

These two are dependent events. If they were independent, then the 54% would have been the same for women and men, but being a woman (or man) changed the overall probability of being in favor of stricter gun control laws.

3. The eight-sector spinner is back. If the spinner is spun once and the outcome is noted answer the following questions.



(a) Let the event S be the event of getting a perfect square, i.e. 1 or 4. What is the probability of getting a perfect square, i.e. $P(S)$?

$$\boxed{P(S)=\frac{2}{8}=\frac{1}{4}}$$

(b) Let E be the event of getting an even. What is the probability of getting a perfect square given you got an even, i.e. $P(S \mid E)$? Are the two events independent? Explain.

$$\boxed{P(S|E)=\frac{1}{4}}$$

These two events are independent because getting an even did not change the probability of getting a perfect square.

(c) Let M be the event of getting a multiple of four. What is the probability of getting a perfect square given that you got a multiple of four, i.e. $P(S \mid M)$? Are the two events independent? Explain.

$$\boxed{P(S|M)=\frac{1}{2}}$$

These two events are not independent. They are not independent because knowing that we spun a multiple of 4 (meaning we could only have gotten a 4 or an 8) changed the probability that we rolled a perfect square from one fourth to one half.

4. If two events, A and B, and independent then $P(\text{A and B})=$

(1) $\dfrac{P(\text{A})}{P(\text{B})}$ (3) $\dfrac{P(\text{B})}{P(\text{A})}$

(2) $P(\text{A})\cdot P(\text{B})$ (4) $P(\text{A})+P(\text{B})$

(2)

5. There is a 34% chance that a person picked at random from the adult population is regular smoker of cigarettes and an 18% chance that a person picked has emphysema. If the percent of the adult population that are both regular smokers and suffer from emphysema is 14%, is being a smoker independent from having emphysema? Justify your result by using the **Product Test for Independence**.

$\boldsymbol{P(\text{S})=0.34}$
$\boldsymbol{P(\text{E})=0.18}$
$\boldsymbol{P(\text{S and E})=0.14}$

$\boldsymbol{0.14=(0.34)(0.18)=0.0612}$

These two evens are <u>not</u> independent. If the events were independent, then the product of the probabilities of S and E would equal the probability of S and E.

6. The two-way frequency table below shows the proportions of a population that have given hair color and eye color combinations. Use this table to answer the following.

(a) Show that the events of having green eyes and red hair are dependent.

$\boldsymbol{P(\text{G})=0.25}$
$\boldsymbol{P(\text{R})=0.18}$
$\boldsymbol{P(\text{G and R})=0.15}$
$\boldsymbol{P(\text{G})\cdot P(\text{R})=(0.25)(0.18)=0.045\neq 0.15}$

		Hair Color			
		Black	Blond	Red	Total
Eye Color	Blue	0.17	0.21	0.02	0.40
	Brown	0.21	0.13	0.01	0.35
	Green	0.07	0.03	0.15	0.25
	Total	0.45	0.37	0.18	1.00

(b) Many of the hair colors have dependence on eye color. Does having blond hair have a dependence on having brown eyes? Show the analysis that leads to your decision.

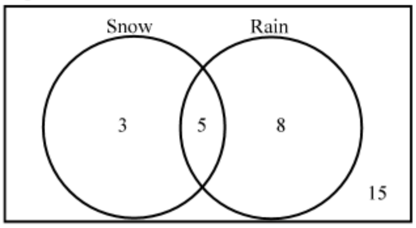
$\boldsymbol{P(\text{blond hair})=0.37}$
$\boldsymbol{P(\text{brown eyes})=0.35}$
$\boldsymbol{P(\text{blond hair and brown eyes})=0.13}$

$\boldsymbol{P(\text{blond hair})\cdot P(\text{brown eyes})=(0.37)(0.35)=0.1295\approx 0.13}$

No, these two do <u>not</u> have a dependence on each other.

7. The month of March has 31 days in it. In New York, March has days when it snows, days when it rains, and days when it does both. This breakdown is shown in the Venn diagram below.

Based on the diagram, are the events of having snow and having rain dependent or independent? Justify.



$\boldsymbol{P(\text{S})=\dfrac{8}{31}\approx 0.26,\ P(\text{R})=\dfrac{13}{31}\approx 0.42,\ P(\text{R and S})=\dfrac{5}{31}\approx 0.16}$

$\boldsymbol{P(S)\cdot P(\text{R})=\dfrac{8}{31}\cdot\dfrac{13}{31}\approx 0.11}$

These two events are slightly dependent on each other.

Probabilities involving **single-stage experiments** are easy enough because only one thing is happening to affect the probability, i.e. you flip a coin once, you pick one person at random, or you pull one card out of a deck. Probabilities, both empirical and theoretical, become increasingly more complicated with **multi-stage experiments**, where more than one thing happens, i.e. you flip a coin three times. How we handle these types of probabilities actually comes from the conditional probability formula.

***Exercise* #1:** Given that the probability of event B occurring given event A has occurred is $P(\text{B} \mid \text{A}) = \dfrac{P(\text{A and B})}{P(\text{A})}$ answer the following.

multiply

Recall: $P(B|A) = P(B)$ when A and B are independent

(a) Rewrite this formula, solving for $P(\text{A and B})$.

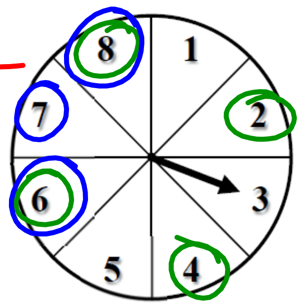
$P(A) \cdot P(B|A) = P(A \text{ and } B)$

(b) How could you write this formula if events A and B were **independent**?

$P(A) \cdot P(B) = P(A \text{ and } B)$

This rearrangement of the conditional probability formula gives us a useful tool for calculating the probability of events that occur in **multi-stage experiments**. You will easily be able to accomplish this if you systematically phrase the questions as unions of events (events connected by AND).

***Exercise* #2:** Consider the spinner shown below. The spinner is spun twice and the result is recorded.



(a) Are the outcomes of the two spins dependent or independent?

B/c first spin has no effect on the second

(b) What is the probability that you will get an even on the first spin and a number greater than five on the second spin?

Even AND >5

$\frac{4}{8} \times \frac{3}{8} = \boxed{\frac{12}{64}} \rightarrow \boxed{\frac{3}{16}}$

(c) What is the probability that you will spin a prime number and a perfect square (in either order)? Note that this is more complex than (b).

* "OR" we <u>add</u>

"AND" we multiply

As experiments grow more complicated with more stages, theoretical probability becomes increasingly more complicated. It is especially important to note whether you are sampling **with or without replacement**.

20 total

Exercise **#3:** A class consists of 12 girls and 8 boys. A group of three is picked to give a speech. If the students are picked at random, what is the probability that they all will be boys? Use the events below to show how you calculated your final answer.

$P(E_1 \text{ and } E_2 \text{ and } E_3)$

Let: E_1 = Event that the first picked was a boy
E_2 = Event that the second picked was a boy
E_3 = Event that the third picked was a boy

$$\frac{8}{20} \times \frac{7}{19} \times \frac{6}{18} = \frac{336}{6840} = \frac{14}{285} \text{ or } 5\%$$

The **multiplication property of probability** is crucial in many applications in engineering decision making.

Exercise **#4:** Say that a power generating facility has three primary safety switches in case of an emergency. The probability that any one of these switches would fail is 5%. What is the probability all three will fail given that the switches are **independent** of one another?

$$P(\text{All Fail}) = (.05)(.05)(.05)$$
$$= .000125 \text{ or } .0125\%$$

Many times when using the multiplication rule we need to be careful about how we frame the question. But, if we properly frame it in terms of AND and OR logical connectors, then the rules of probability will work out.

Exercise **#5:** A company was determining the effectiveness of its warranty sales on computers. They took data on the number of customers who purchased warranties on two different brands of computers. If a customer was chosen at random, what is the probability they did not purchase a warranty?

Type1 with no warranty

$P(1 \text{ and no warranty})$

$= (.68)(.65) = .442$

	Percentage of Customers Purchasing	Percent of Those Who Purchased that Also Purchased Warranty
Type 1	68%	35% 65% didn't
Type 2	32%	56% 44% didn't

Type 2 with no warranty

$P = (.32)(.44) = .1408$

Type1 with no warranty OR Type 2 with no warranty

$P = .442 + .1408$

$= .5828$

$\approx 58\%$

Independent: one event has no effect on the outcome of another.

Mutually exclusive: the occurance of one outcome excludes all other outcomes.

ex: Head and Tail on a coin are mutually exclusive because they can't happen at the same time.

Test Tues 9/19

HW: Review Sheet